ME 315
Theory of Machines – Design of Elements
Fall 2015

Design Project

Group members

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Introduction

The goal of this project is to design a gear train that takes an input of 7000W in Shaft 1 rotating at 2000 rpm and transfers 800 rpm and 650 rpm to Shafts 2 and 3 respectively. The gear train system is assumed to be 100% efficient, has a reliability of 99% and is expected to work for 5 years (5 days a week, 52 weeks a years and 8 hours a day). Figure 1 below shows the gear train system.

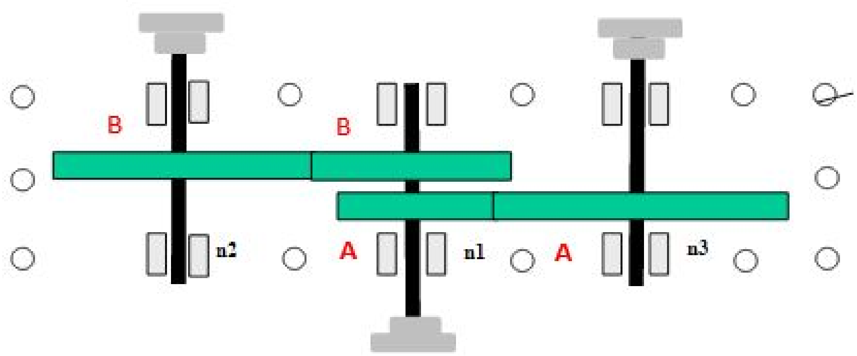


Figure 1: Gear train to be designed

Tables 1 and 2 below show the various parameters of each of the gears. Note that the number of teeth per gear was chosen such that one gear in each pair would have a prime number of teeth, and such that the output speed was within 5rpm of the project targets. Table 3 shows the bearing parameters for each shaft. Analyses of each gear and shaft follows this.

Table 1: Gear Train Specifications

Gear #	# of teeth	Module (mm)	Desired speed (rpm)	Actual Speed (rpm)	Error (rpm)
1(A)	33	3	2000	2000	∓0
2(A)	83	3	800	795.18	-4.84
3(B)	23	3	2000	2000	∓0
4(B)	71	3	650	647.89	-2.11

Table 2: Gear Geometries

Gear #	Pitch Diameter (mm)	Pitch Angle	Addendum, a (mm)	Dedendum, b (mm)	Face Width, bw (mm)
1(A)	99	20	3	3.75	38
2(A)	249	20	3	3.75	36
3(B)	69	20	3	3.75	38
4(B)	213	20	3	3.75	36

Table 3: Bearing parameters

Shaft	Bearing Number	Inner Diameter (mm)	Outer Diameter (mm)	Width (mm)	Basic Load Rating C (N)	Basic Load Rating C_0 (N)	Life (millions of cycles)
Shaft 1	6202	15	35	11	7800	3750	1248
Shaft 2	16004	20	42	8	6890	4050	496.2
Shaft 3	16004	20	42	8	6890	4050	404.3

Shaft Assembly Drawings

Below are picture of the shaft assemblies for all shafts. The following pages show the drawings of each shaft assembly, the shaft itself and a gear from the assembly. In the case of Shaft 1, only one gear is shown.

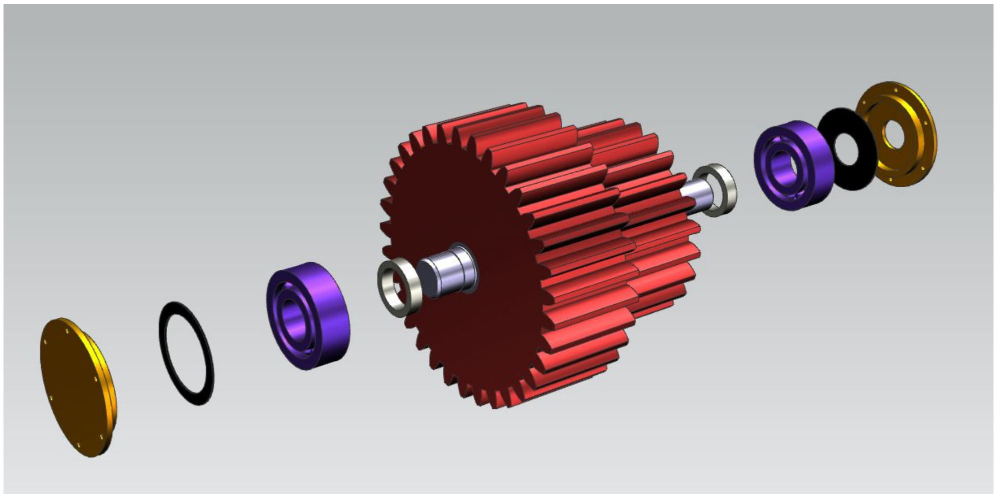


Figure 1: Shaft 1 Assembly

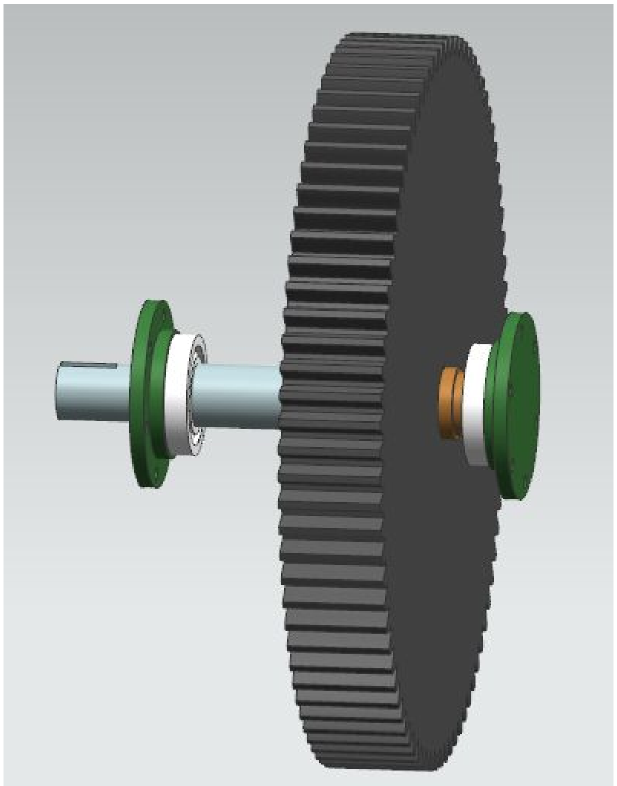


Figure 2: Shaft 2 Assembly

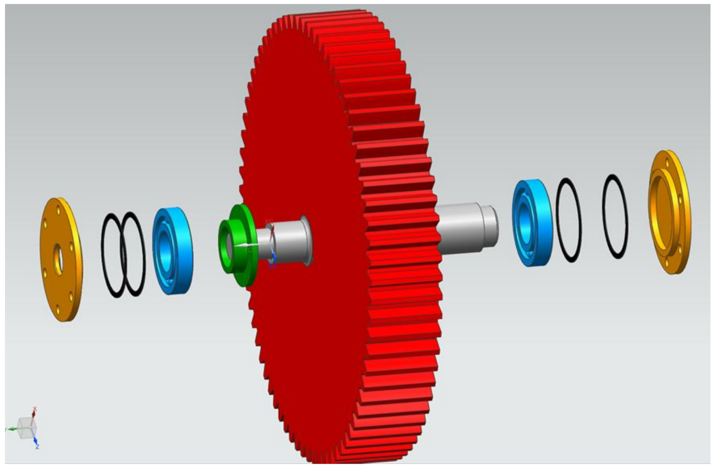


Figure 3: Shaft 3 Assembly

Appendix 1: Analysis of Shaft System 1

The pinions are designed to be made from through-hardened steel with a Young's Modulus of 200 GPa, a yield strength of 250 MPa, an ultimate tensile stress of 800 MPa and a Poisson's ratio of 0.3. A hardness of 300HB was chosen for the pinions as well as a grade of 2. The shaft will also be made of the same material. It was assumed that one gear in the pair of gears in the gear reducer would have full power transmitted through it. Therefore, the gears and shaft were designed assuming that all the 7000 W of power would be transmitted through it at some point.

Design of Gears

<u>Geometry of gear</u>

Using the equation $d = mN$ and the values listed in Table 1, the diameter for Pinion A is 69 mm and the diameter of Pinion B is 99 mm. The face width is found using the equation $b = 12m$ which gives 36 mm for both pinions. However, 2 mm is added to on this to give a face width of 38 mm so that the face widths of the pinions is larger than those of the gears. The pressure angle ϕ of each pinion is 20.$^{\circ}$

<u>Forces on gears</u>

The torque acting on each pinion is

$$T = \frac{60H}{2\pi n}$$

Where H equals 7000 watts.

The tangential force and radial force on each pinion are respectively

$$W^t = \frac{T}{d/2} \; and \quad W^r = W^t \tan\phi$$

The torques and forces on each pinion are summarized in the table below.

	Torque T (N)	**Tangential force W^t (N)**	**Radial force W^r (N)**
Pinion A	33.42	968.77	352.60
Pinion B	33.42	675.20	245.75

<u>Allowable stresses of gears</u>

To find the allowable stresses of each pinion, the allowable contact stress and bending stress before modifications must be found first. The allowable bending stress before modifications is found with the equation $S_b = 0.703HB + 113$ which gives a stress of 323.9 MPa while the contact stress is found using the equation $S_c = 2.41HB + 237$ which gives a stress of 960 MPa.

The allowable bending stress of the pinion is given by the equation

$$\sigma_{b,all} = \frac{S_b Y_N}{K_T K_R}$$

And the allowable contact stress of the pinion is found using the equation

$$\sigma_{c,all} = \frac{S_c Z_N}{K_T K_R}$$

where K_T, the temperature factor, is 1 while K_R, the reliability factor is also 1 at a 99% chance of survival. Y_N, the bending stress cycle factor is found using the equation

$$Y_N = 1.6831 N^{-0.0323}$$

And Z_N, the contact stress cycle factor is found using the equation

$$Z_N = 2.466 N^{-0.056}$$

N represents the number of cycles the shaft must sustain for the lifetime stated previously is

$$N = 5yrs * \frac{52\ weeks}{yr} * \frac{5\ days}{week} * \frac{8hr}{day} * \frac{60min}{hr} * \frac{2000rev}{min} = 1.248 * 10^9\ cycles$$

Using all these equations, $\sigma_{b,all}$ and $\sigma_{c,all}$ can be calculated and are found in the table below.

	Allowable bending stress (MPa)	**Allowable contact stress (MPa)**
Pinion A	274.1	718.8
Pinion B	274.1	718.8

The results are the same because both pinions have the same material properties and operate under the same conditions.

<u>Working stress of gears</u>

In order to find the factor of safety of the pinions, their working stresses must also be found. The bending stress is given by

$$\sigma_b = \frac{W^t P}{b_w Y_j} K_a K_s K_m K_v K_i K_b$$

And the contact stress is given by

$$\sigma_c = K_e \left(\frac{W^t}{b_w d_p} \frac{1}{I} K_a K_s K_m K_v\right)^{1/2}$$

Before the bending and contact stresses can be calculated, the various stress factors must be found. The table below gives the values of the stress factors used.

Stress factor	**Pinion A**	**Pinion B**
Rim thickness factor, K_b	1.00	1.00
Idler factor, K_i	1.00	1.00
Dynamic factor, K_v	3.41	3.41
Load distribution factor, K_m	1.16	1.22
Size factor, K_s	1.00	1.00
Application factor, K_a	1.00	1.00
Geometry factor Y_j	0.36	0.41
Geometry factor I	0.7626	0.7224
Elasticity factor, K_e	480380	480380

K_v, K_e and I were calculated using equations and were not just read off a table or a graph.

K_v is found using the equation

$$K_v = ((A + \sqrt{200V})/A)^B$$

Where

$$V = \frac{\pi(2000rpm)}{60} = 209.45\ m/s$$

$$A = 50 + 56(1 - B)$$

$$B = \frac{(12-6)^{\frac{2}{3}}}{4}.$$

K_e is found using the equation

$$\sqrt{\frac{1}{1 - v^2/E}}$$

Where E is the Young's modulus of the gear and v is the Poisson's ratio.

Finally, I is found using the equation

$$\frac{\pi\cos(\phi)\sin(\phi)}{1+{d_p}/{d_g}}$$

Where d_p is the diameter of the pinion and d_g is the diameter of their respective meshing gear. The diameters of Gear A and Gear B are known to be 213 mm and 249 mm respectively and these values were used to find the value of I.

These values, as well as the forces and dimensions found earlier, can then be used to solve for the bending and contact stresses of each pinion, the result of which can be found below.

	Bending stress (MPa)	**Contact stress (MPa)**
Pinion A	93.4	649.3
Pinion B	60.1	476.8

Factors of safety of gears

In general, the factor of safety of a part is

$$n_s = \frac{\sigma_{allowed}}{\sigma_{design}}$$

This can be used to find the factors of safety of each pinion. The factors of safety for each of the stresses are summarized in the table below.

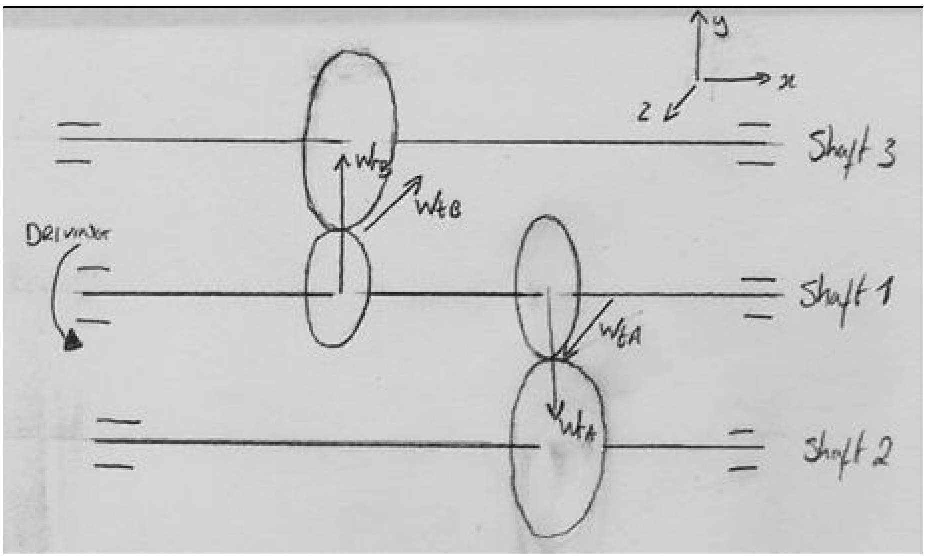
	Factor of safety bending stress	**Factor of safety contact stress**
Pinion A	2.93	1.11
Pinion B	4.56	1.51

From the table, it is clear that Pinion B is the weakest part of this shaft as it has the lowest factor of safety under contact stress.

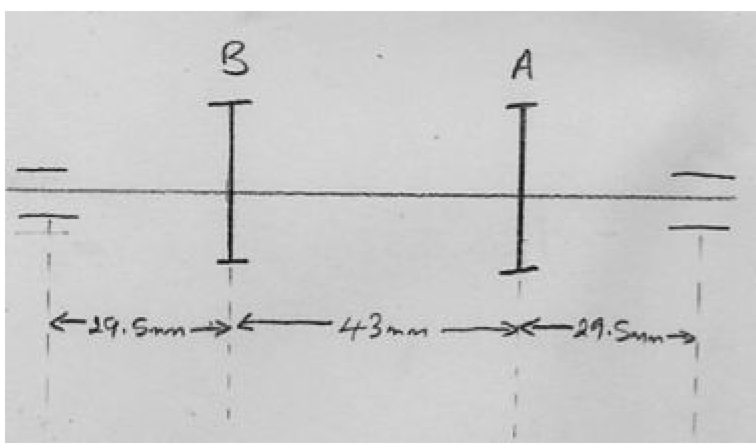
Design of Shaft

Geometry of shaft

Two bearings and two gears will be placed on Shaft 1. There will be a 5 mm clearance between the bearing and the gear and that there will be a 5 mm clearance between the two gears. The shaft is 113 mm in length with a mid-bearing to mid-bearing length of 102 mm. The figures below show a simple sketch of the shaft system as well as a sketch of Shaft 1 with relevant lengths.



Simple sketch of shaft system



Simple sketch of Shaft 1

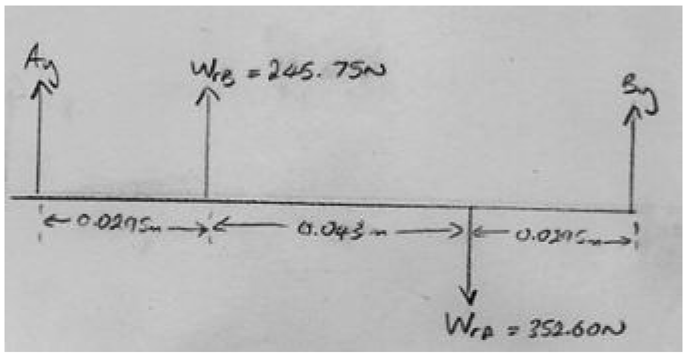
The diameter of the shaft must also be found. The minimum diameter is found using the equation

$$d = \sqrt[3]{\frac{16T}{\pi\tau_{all}}} \text{ where } \tau_{all} = \frac{0.577S_y}{n_s}$$

The factor of safety n_s should be at least 1.1. S_y is the yield strength which is 200 MPa and T is the torque calculated previously. It is best to overdesign the shaft and reduce its diameter if necessary so the factor of safety for the equation above is taken to be 2. This gives a diameter of 14.34 mm. However, the gears on the shaft will have a keyway and so the shaft diameter must be increased by 5% to 15.05 mm. The diameter of the shaft was rounded down and taken to be 15.0 mm.

Forces and moments on shafts

The diagram below shows the radial forces acting on the shaft.



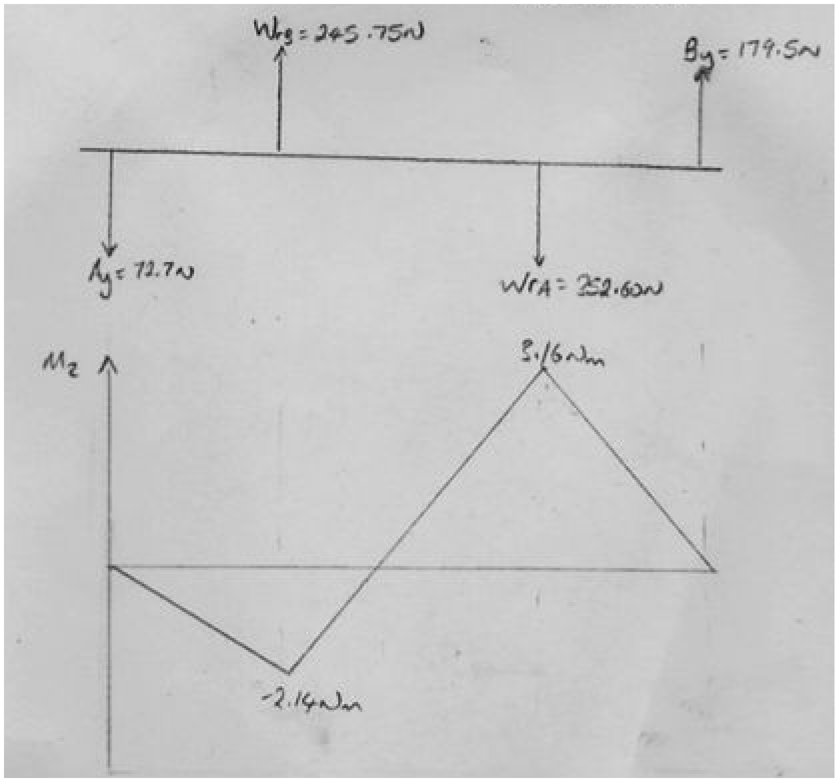
Taking the sum of moments about point A, the resultant force at B, B_y, can be calculated from

$$B_y = \frac{W_A^r(a+b) - W_B^r(a)}{a+b+c}$$

And A_y is calculated by taking the sum of forces as zero

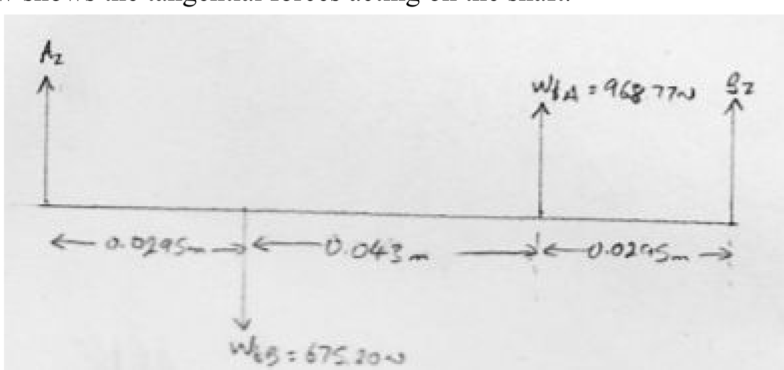
$$A_y = W_A^r - W_B^r - B_y$$

This gives B_y as 179.5 N and A_y as -72.7 N (the negative sign indicating that it is in the opposite direction to what was indicated). Since B_y is larger than A_y, the maximum moment is given by $M_{max} = B_y(c)$ which equals 5.30 Nm. The free body diagram and moment diagram in the xy plane is below.



Free body diagram and moment diagram for radial forces

The diagrams below shows the tangential forces acting on the shaft.

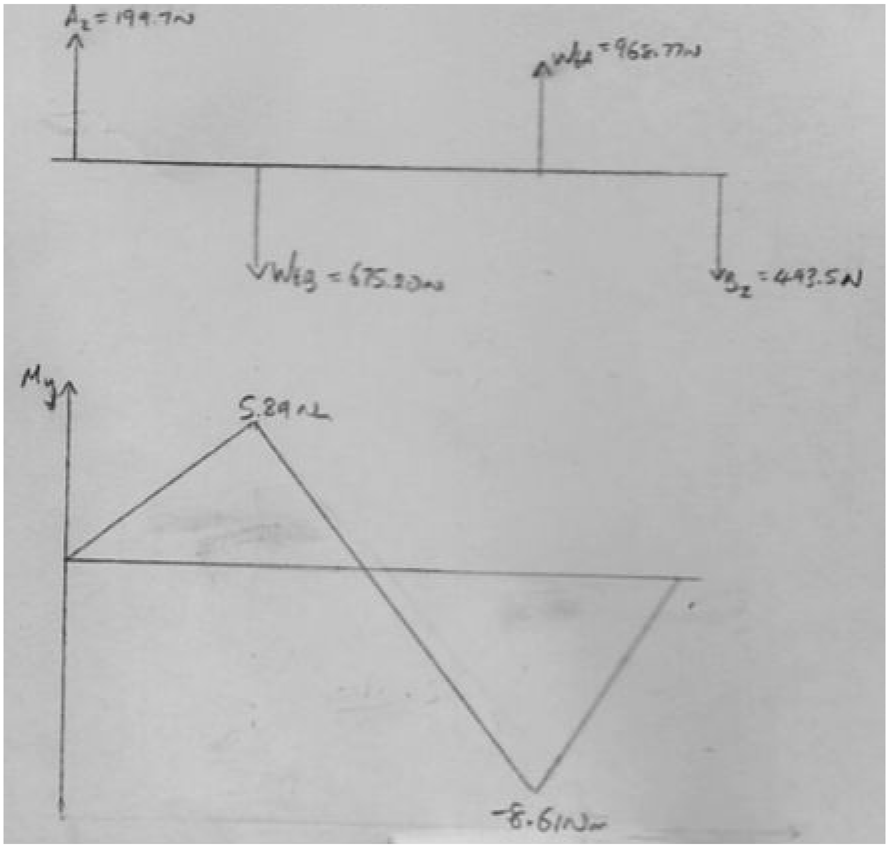


A similar process is used to find A_z, B_z and M_{max}.

$$B_z = \frac{W_B^t(a) - W_A^t(a+b)}{a+b+c}$$

$$A_z = W_B^t - W_A^t - B_z$$

This gives B_z as -493.3 N and A_z as 199.7 N. Therefore, $M_{max} = B_z(c)$ which equals -14.5 Nm. The free body diagram and moment diagram in the xz plane is below.



Free body diagram and moment diagram for tangential forces

The resultant moment can then be calculated from M_{max} of both planes

$$M_{res} = \sqrt{M_{x,max} + M_{y,max}}$$

Which gives a value of 15.5 Nm. The critical point is at point A where the larger pinion is located.

<u>Working stresses of shaft</u>

The bending stress and shear stress acting on the beam will determine the Von Mises stresses and hence the factor of safety of the shaft. The bending stress on the shaft is given by the equation

$$\sigma_b = \frac{32M_{res}}{\pi d^3}$$

Using the resultant moment M_{res} of 15.5 Nm and the diameter of 15.0 mm, the bending stress is calculated to be 46.7 MPa.

The shear stress is given by the equation

$$\frac{16T}{\pi d^3}$$

Using the diameter of 15.0 mm and the torque calculated previously of 33.42 N, the shear stress is calculated to be 50.4 MPa.

The Von Mises stress amplitude is defined as

$$\sigma_{von-a} = \sqrt{\sigma_{ba}{}^2 + 3(\tau_a)^2}$$

Which gives a value of 46.7 MPa when using as the bending stress calculated previously.

The Von Mises mid-range amplitude is defined as

$$\sigma_{von-m} = \sqrt{\sigma_{bm}{}^2 + 3(\tau_m)^2}$$

Which gives a value of 87.4 MPa when using as the shear stress calculated previously. These working stresses can be used to find the factor of safety of the shaft.

Factor of safety of shaft

To find the factor of safety, the endurance limit S_e must first be found. S_e is calculated using the equation

$$S_e = k_f k_s k_r k_t k_m S_e' \quad where\ S_e' = 0.5 S_{ut}\ for\ bending$$

The ultimate tensile is 800 MPa. The table below gives the values of the stress factors used.

Stress factor	Equation/Assumption	Value
Surface finish factor k_f	$e(S_{ut})^f = 1.58(800)^{-0.085}$	0.895
Size factor k_s	$1.189(d)^{-0.112} = 1.189(15)^{-0.112}$	0.878
Reliability factor k_r	99% reliability	0.820
Temperature factor k_t	Room temperature	1
Miscellaneous factor k_m	No miscellaneous factors	1

Using the equation above and the stress factors, a value of 257.7 MPa is calculated as the endurance limit.

The factor of safety of the shaft should be calculated using the Goodman line and the Yield line. The Goodman line for designing against fatigue failure is

$$\frac{\sigma_{von-a}}{S_e} + \frac{\sigma_{von-m}}{S_{ut}} = \frac{1}{n_s}$$

Using the values calculated previously, a value of 3.44 is obtained as the factor of safety n_s.

The Yield line for designing against fatigue failure is

$$\frac{\sigma_{von-a}}{S_y} + \frac{\sigma_{von-m}}{S_y} = \frac{1}{n_s}$$

Using the values calculated previously and a yield stress of 200 MPa, a value of 1.49 is obtained as the factor of safety.

The factors of safety for each of the criteria are summarized in the table below.

Criteria	**Factor of safety**
Goodman line	3.44
Yield line	1.49

Bearing Selection

The radial force acting on the bearing was determined using the resultant forces acting on the 113 mm shaft. The radial forces at both ends of the shaft by finding the resultant of forces in the y and z directions. The largest radial force was determined to be 536.1 N.

The L10 life of the shaft required is 10400 hours. Using the equation

$$L_{10} = \frac{h(60)n}{10^6}$$

Where h is the L10 life in hours and n is the speed of the shaft (2000 rpm). The L10 life in millions of cycles was calculated to be 1248 million cycles.

The initial bearing rating C was calculated using the equation

$$C = PL^{\frac{1}{3}}$$

Where L is the L10 life in millions of cycles and P is the force acting on the bearing. The use of spur gears in the gear reducer means that there is no axial force acting on the bearings. Therefore, P can be considered to be the highest radial force which is 536.1 N. This gives a bearing rating C of 5652.0 N. Using Table 6-3 in the text book and considering the minimum diameter to be 20 mm, we obtain a basic load rating C of 7800 N and a C_o value of 3750 N.

Normally, the value of P is then recalculated using the equation

$$P = XP_r + YP_a$$

Where X and Y are radial load factors, P_a is the axial force and P_r is the radial force. However, in this case, P_a is 0 which gives X a value of 1 and Y a value of 0. The P value remains the same so the bearing selection is complete.

The bearing selected is SKF 6202. The nominal diameter of the bearing is 15 mm, the outer diameter is 35 mm and the width is 11 mm.

Appendix 2: Analysis of Shaft System 2

The assumptions used for analysis are listed here. Following these assumptions is the full handwritten analysis of the shaft, gear, and bearing.

Most of the values calculated in this analysis were calculated using MATLAB code, which allowed several parameters (namely the modulus of the gear and the diameter of the shaft) to vary in some capacity. These minimized values are also reported here.

The bending moment/torque diagram for the shaft was also generated using MATLAB and is reported here before the handwritten calculations.

Assumptions:

General

Power = 7kW (If the other output is disconnected [worst case], and the power remains the given 7kW, then power through this shaft must be the full 7kW)

Shaft Dimensions: 47mm from bearing to gear in input side of the assembly, 114mm from bearing to gear on output side of housing. Distances are measured center-to-center.

Force distribution: Forces are applied on the shaft at the center of each gear or bearing

Reliability: 99%, for all components

Ambient Temperature: Negligible in effect

Shaft properties

Ultimate Tensile Strength (Shaft material): 800MPa (Taken from a homework problem, assumed to be representative of steel)

Yield Strength (Shaft material): 500MPa, (Also taken from a homework problem, assumed to be representative of steel)

Shaft finish: the finish correction factor was chosen as .8, which based on rough calculations corresponds to a level of finishing between machined and ground.

Shaft Shoulders: Ignored for this analysis. It is assumed that the stress concentration factor at a shoulder will be less than 2 for a reasonable fillet size. Because of this, shaft safety factor was only accepted at a minimum of 2(1.1)=2.2. This allows for the shaft shoulder dimensions to be set later, once all other components are selected. This will be analysed to confirm safety after this is set.

Torque application: For the purpose of analysis of the worst-case, torque is assumed to apply cyclically at the same period as the bending stress, which cycles per rotation. In actuality, torque would generally only cycle once per on/off cycle, typically once per day.

Gear Properties

Gear Quality: 6 (out of 12; corresponds to high-quality commercial gears)

Young's Modulus: 210GPa (Also taken from a homework problem, assumed to be representative of steel)

Poisson's Ratio: 0.3 (Also taken from a homework problem, assumed to be representative of steel)

Bearing Properties

Bearings were chosen to use the same bearing at either support.

Variables chosen

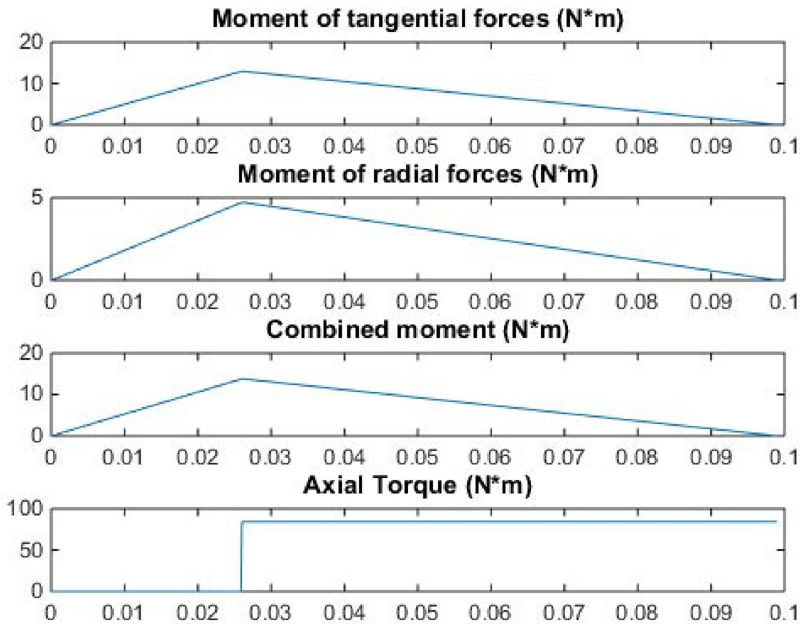
These values were allowed to vary during analysis. The resultant value is considered a reasonable, conservative minimum for size

Module = 3mm (Note: Lower modules were acceptable for this particular gear, but the central shaft system required 3mm as a minimum possible value)

Shaft Diameter = 20mm

Factors of Safety

Yield line safety factor (shaft)	5.12
Goodman line safety factor (shaft)	4.96
Tooth bending safety factor (gear)	7.52
Contact stress safety factor (gear)	3.44



Bending Moment and Torque diagram for shaft system 2